A NEW PROCEDURE FOR DESIGNING SINGLE SAMPLING PLAN INDEXED THROUGH TRIGNOMETRIC RATIOS

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ABSTRACT

Acceptance Sampling is the methodology which deals with procedures through which decisions to accept or reject of a lot which are based on the result of inspection of samples. The foundation of the scheme of acceptance sampling has laid by Dodge and Romig (1959). General procedures and necessary tables are provided for the selection of single sampling plan through tangent angle as proposed by Norman Bush (1953). Mandelson (1962) has explained the desirability for developing a system of sampling plans indexed through MAPD. Mayer (1967) has explained that the quality standard that the MAPD can be considered as a quality level with other conditions to specify the OC curve. Soundararajan (1975) has constructed tables for selection of single sampling plan indexed through MAPD and K (p_T/p_*) Suresh and Ramkumar (1996) have studied the selection of single sampling plan indexed through Maximum Allowable Average Outgoing Quality (MAAOQ) and MAPD.

This paper provides a new procedure for designing a single sampling plan indexed through trigonometric ratios, hypotenuse ratios along with decision region (d_1) and probabilistic region (d_2) which is more applicable in practical situations. Numerical illustrations are also provided for the construction and selection of the plan parameters using trigonometric ratios and hypotenuse ratios.

Key Words: *Single Sampling Plan, Decision Region, Operating Characteristic Curve, Trigonometric ratios*

Introduction

Acceptance Sampling is defined as the procedure for inspection and classification of sample of units selected at random from a larger lot and ultimate decision about the disposition of the lot is made. Basically the “acceptance quality control” system that was developed encompasses the concept of protecting the consumer from getting unacceptable defective product, and encouraging the producer in the use of process quality control through varying the quantity and severity of acceptance inspections in direct relation to the importance of the characteristics inspected, and the inverse relation to the goodness of the quality level as indication of those inspections.

The single sampling plan is the most widely used basic sampling plan in the area of acceptance sampling. The performance of a sampling plan is identified through an OC curve. For designing a sampling inspection plan, it is the usual practice to consider the OC curve passes through any two of the quality levels. Mandelson (1962) has explained the desirability for developing such a system of sampling plans indexed through MAPD Mayer (1967) has explained that the quality standard that the MAPD can be considered as a quality level along with other conditions to specify an OC curve. Soundararajan (1975) has constructed tables for selection of single sampling plan indexed through MAPD and

K= $\frac{p_T}{p_*}$ .Suresh and Ramkumar (1996) have studied the selection of single sampling plan indexed through Maximum Allowable Average Outgoing Quality (MAAOQ).

This paper provides a new procedure for designing attribute single sampling plan indexed through trigonometric ratios and hypotenuse ratios. Also considering the ability of the declination angles of the tangent at the inflection point on the OC curve for discrimination of the Single Sampling Plan (SSP)

Here, $\tan\theta_1 = \frac{0.95 - L(p_*)}{d_1}$ ……………………… (1)

From (1) one can find (n,c) for a particular L(p*) and d_1.So we can state that both θ_1 and d_1 uniquely determines the SSP.

Similarly, $\tan\theta_2 = \frac{L(p_*) - 0.10}{d_2 - d_1}$ ………………… (2)

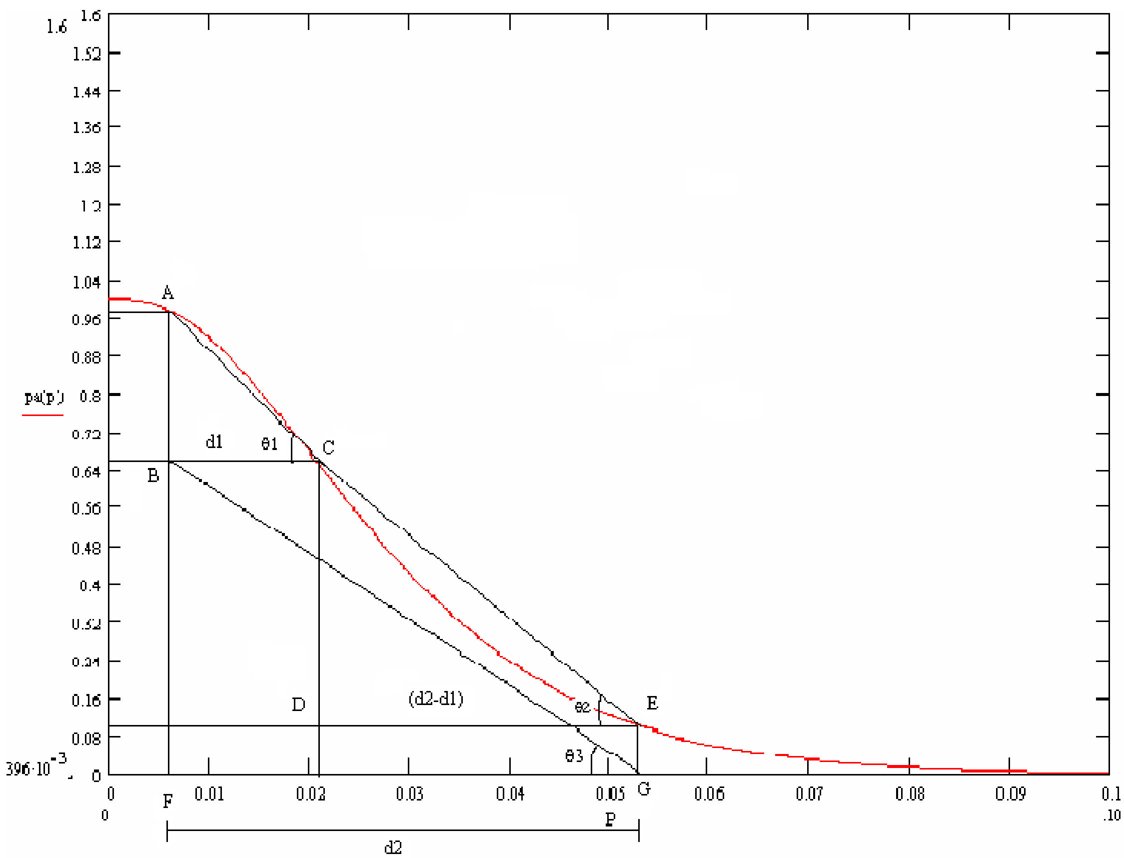
From (2) one can find (n,c) for a particular L(p*) and (d_2-d_1). So we can state that both θ_2 and (d_2-d_1) uniquely determines the SSP.

And, $\tan\theta_3 = \frac{L(p_*)}{d_2}$ …………………………… (3)

From (3) one can find (n,c) for a particular L(p*) and d_2. So we can state that both θ_3 and d_2 uniquely determines the SSP.

From figure1, we have ΔABC represents the approximate area inscribed by the quality levels p_1 and p_*. ΔCDE represents the approximate area inscribed by the quality levels p_* and p_2.And the ΔBFG represents the approximate area inscribed by the quality levels p_1 and p_2. θ_1 is the inscribed triangle by OC with quality levels p_1 and p_*. θ_2 represent the inscribed triangle by OC with quality levels p_* and p_2. And θ_3 is the inscribed triangle by OC with quality levels p_1 and p_2.

Fig 1: OC Curve for Decision Region (d1) and Probabilistic Region (d2) and Tangent Angles



Selection of sampling plans

Table 1 is given for selected values of c. Here SSP with c=0 are not considered, since c=0 plans do not involve an inflection point on the OC curve. Tables are given for the values of $L(p_*)$ for c=1,2,……20.

Table 1: Certain Parametric Values for SSP

c	L(p*)	np_1	np_2	d_1	d_2
1	0.735759	0.355	3.89	0.645	3.535
2	0.676676	0.818	5.322	1.182	4.504
3	0.647232	1.366	6.681	1.634	5.315
4	0.628837	1.97	7.994	2.03	6.024
5	0.615961	2.613	9.275	2.387	6.662
6	0.606303	3.286	10.532	2.714	7.246
7	0.598714	3.981	11.771	3.019	7.79
8	0.592547	4.695	12.995	3.305	8.3
9	0.587408	5.426	14.206	3.574	8.78
10	0.58304	6.169	15.407	3.831	9.238
11	0.579267	6.924	16.598	4.076	9.674
12	0.575965	7.69	17.782	4.31	10.092
13	0.573045	8.464	18.958	4.536	10.494
14	0.570437	9.246	20.128	4.754	10.882
15	0.56809	10.035	21.292	4.965	11.257
16	0.565962	10.831	22.452	5.169	11.621
17	0.564023	11.633	23.606	5.367	11.973
18	0.562245	12.442	24.756	5.558	12.314
19	0.560607	13.254	25.902	5.746	12.648
20	0.559093	14.072	27.045	5.928	12.973

Using the table it can be noted that as c increased d_1, d_2 increases but $L(p_*)$ decreases.

Example 1

For a given sample size n=100 and to attain an area of 0.85.Find the acceptance to be taken for attain a better OC curve.

Using table 2 we can easily read off, for area ABC=0.8549 the corresponding acceptance number c=13.

Example 2

For a given sample size n=100 and to attain an area of 1.18, find the acceptance to be taken for attain a better OC curve.

Using table 3 we can find that for area CDE=1.18, the corresponding acceptance number c=7.

Example3

For a given sample size n=100 and to attain an area of 3, find the acceptance to be taken for attain a better OC curve.

Using table 4 we can easily read off, that the appropriate acceptance number is c=13.

Example 4

For an OC curve to which a tangent is drawn,it is specified the operating ratio R_4=2.2394. Find the appropriate acceptance number.

Using table 8 it can be seen that the appropriate c for the operating ratio R_4=2.2394 is 3.That is c=3.

Construction of Tables

When the proportion of defective in the lot is small and sample size is large so that np<5 then the lot quality assumed to follow Poisson distribution. The probability of acceptance under Poisson model is given as

$$L(p) = \sum_{r=0}^{c} \frac{e^{-np}(np)^r}{r!} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(4)}$$

Where p is the proportion defective of the lot, p coordinate of the inflection point will obtain as $p_* = \frac{c}{n}$.

$L(p_*)$ represents the probability of acceptance of an utmost satisfactory quality (MAPD)

$$L(p_*) = \sum_{r=0}^{c} \frac{e^{-c}(c)^r}{r!} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \text{(5)}$$

Thus $L(p_*)$is a function of c alone, and it is constant for fixed c.

From figure, $\tan\theta_1 = \frac{0.95 - L(p_*)}{d_1}$, the declination angle $\theta_1 = \tan^{-1}\left(\frac{0.95 - L(p_*)}{d_1}\right)$.

Similarly $\tan\theta_2 = \frac{L(p_*) - 0.10}{d_2 - d_1}$ and the declination angle is $\theta_2 = \tan^{-1}\left(\frac{.L(p_*) - 0.10}{d_2 - d_1}\right)$.

And $\tan\theta_3 = \frac{L(p_*)}{d_2}$, the declination angle is $\theta_3 = \tan^{-1}\left(\frac{.L(p_*)}{d_2}\right)$.

For different values of c=1,2,…..20, $L(p_*)$ is determined from equation (5). Substituting the appropriate values in equation (1),(2),(3) for fixed $L(p_*)$,d_1,
(d_2-d_1), d_2 and hence angle $\theta_1, \theta_2, \theta_3$ and (n,c) are obtained.

Table-2 provides the area of triangle ABC for a fixed n for different values of c. Table-3 provides the area of triangle CDE for a fixed n for different values of c. Table-4 provides the area of triangle BFG for a fixed n for different values of c Table-5 provides the operating ratio R_1 for different values of c. Table-6 provides the operating ratioR_2 for different values of c. Table-7 provides the operating ratio R_3 for different values of c. Table-8 provides the operating ratio R_4.

Conclusion

MAPD is the quality measure proposed for designing the sampling plan. MAPD has evolved as a world wide accepted quality measure to discriminate between good and bad lots. Many procedures for designing single sampling plan have been developing over years using MAPD as quality index. When sampling procedure fails to obtain OC curve which lies closer to the ideal one. MAPD related plans which are more efficient for achieving better quality products. Therefore Quality parameters like trigonometric ratio's, hypotenuse ratio's decision region (d_1), probabilistic region (d_2) which are more applicable in suitable situations.

Table 2: The area of triangle ABC for a fixed n

c	d_1	L(p*)	.95-L(p*)	AC	areaABC
1	0.645	0.7358	0.2142	0.6797	0.0691
2	1.182	0.6767	0.2733	1.2132	0.1615
3	1.634	0.6472	0.3028	1.6618	0.2474
4	2.03	0.6288	0.3212	2.0552	0.3260
5	2.387	0.6160	0.3340	2.4103	0.3987
6	2.714	0.6063	0.3437	2.7357	0.4664
7	3.019	0.5987	0.3513	3.0394	0.5303
8	3.305	0.5925	0.3575	3.3243	0.5907
9	3.574	0.5874	0.3626	3.5923	0.6480
10	3.831	0.5830	0.3670	3.8485	0.7029
11	4.076	0.5793	0.3707	4.0928	0.7556
12	4.31	0.5760	0.3740	4.3262	0.8060
13	4.536	0.5730	0.3770	4.5516	0.8549
14	4.754	0.5704	0.3796	4.7691	0.9022
15	4.965	0.5681	0.3819	4.9797	0.9481
16	5.169	0.5660	0.3840	5.1832	0.9925
17	5.367	0.5640	0.3860	5.3809	1.0358
18	5.558	0.5622	0.3878	5.5715	1.0776
19	5.746	0.5606	0.3894	5.7592	1.1187
20	5.928	0.5591	0.3909	5.9409	1.1586

Table 3: The area of triangle CDE for a fixed n

c	d2-d1	L(p*)	L(p*)-.10	CE	area CDE
1	2.89	0.73576	0.6358	2.9591	0.9187
2	3.322	0.67668	0.5767	3.3717	0.9579
3	3.681	0.64723	0.5472	3.7215	1.0072
4	3.994	0.62884	0.5288	4.0289	1.0561
5	4.275	0.61596	0.5160	4.3060	1.1029
6	4.532	0.6063	0.5063	4.5602	1.1473
7	4.771	0.59871	0.4987	4.7970	1.1897
8	4.995	0.59255	0.4925	5.0192	1.2301
9	5.206	0.58741	0.4874	5.2288	1.2687
10	5.407	0.58304	0.4830	5.4285	1.3059
11	5.598	0.57927	0.4793	5.6185	1.3415
12	5.782	0.57597	0.4760	5.8016	1.3760
13	5.958	0.57304	0.4730	5.9767	1.4092
14	6.128	0.57044	0.4704	6.1460	1.4414
15	6.292	0.56809	0.4681	6.3094	1.4726
16	6.452	0.56596	0.4660	6.4688	1.5032
17	6.606	0.56402	0.4640	6.6223	1.5327
18	6.756	0.56224	0.4622	6.7718	1.5615
19	6.902	0.56061	0.4606	6.9174	1.5896
20	7.045	0.55909	0.4591	7.0599	1.6172

Table 4: The area of triangle BFG for a fixed n

c	L(p*)	d2	FG	area BFG
1	0.7358	3.535	3.611	1.300
2	0.6767	4.504	4.555	1.524
3	0.6472	5.315	5.354	1.720
4	0.6288	6.024	6.057	1.894
5	0.6160	6.662	6.690	2.052
6	0.6063	7.246	7.271	2.197
7	0.5987	7.79	7.813	2.332
8	0.5925	8.3	8.321	2.459
9	0.5874	8.78	8.800	2.579
10	0.5830	9.238	9.256	2.693
11	0.5793	9.674	9.691	2.802
12	0.5760	10.092	10.108	2.906
13	0.5730	10.494	10.510	3.007
14	0.5704	10.882	10.897	3.104
15	0.5681	11.257	11.271	3.197
16	0.5660	11.621	11.635	3.289
17	0.5640	11.973	11.986	3.377
18	0.5622	12.314	12.327	3.462
19	0.5606	12.648	12.660	3.545
20	0.5591	12.973	12.985	3.627

Table 5: The ratio of area of triangle ABC and CDE

c	area ABC	area CDE	R_1=CDE/ABC
1	0.0691	0.9187	13.2962
2	0.1615	0.9579	5.9297
3	0.2474	1.0072	4.0717
4	0.3260	1.0561	3.2397
5	0.3987	1.1029	2.7663
6	0.4664	1.1473	2.4599
7	0.5303	1.1897	2.2436
8	0.5907	1.2301	2.0825
9	0.6480	1.2687	1.9581
10	0.7029	1.3059	1.8578
11	0.7556	1.3415	1.7755
12	0.8060	1.3760	1.7071
13	0.8549	1.4092	1.6483
14	0.9022	1.4414	1.5976
15	0.9481	1.4726	1.5532
16	0.9925	1.5032	1.5145
17	1.0358	1.5327	1.4797
18	1.0776	1.5615	1.4491
19	1.1187	1.5896	1.4209
20	1.1586	1.6172	1.3957

Table 6: The ratio of area of triangle ABC and BFG

c	area ABC	area BFG	R_2=BFG/ABC
1	0.069093	1.300454	18.8219
2	0.161534	1.523875	9.4338
3	0.247362	1.720019	6.9535
4	0.325981	1.894057	5.8103
5	0.398676	2.051765	5.1464
6	0.466397	2.196635	4.7098
7	0.530266	2.33199	4.3978
8	0.590691	2.459071	4.1630
9	0.647951	2.578722	3.9798
10	0.702912	2.693061	3.8313
11	0.755554	2.801913	3.7084
12	0.806045	2.906321	3.6057
13	0.854935	3.006765	3.5170
14	0.902222	3.103746	3.4401
15	0.948093	3.197492	3.3726
16	0.992545	3.288525	3.3132
17	1.03577	3.376523	3.2599
18	1.077571	3.461742	3.2125
19	1.118725	3.545281	3.1690
20	1.15865	3.626554	3.1299

Table 7: The ratio of area of triangle CDE and BFG

c	area CDE	area BFG	R_3=BFG/CDE
1	0.9187	1.3005	1.4156
2	0.9579	1.5239	1.5909
3	1.0072	1.7200	1.7078
4	1.0561	1.8941	1.7935
5	1.1029	2.0518	1.8604
6	1.1473	2.1966	1.9146
7	1.1897	2.3320	1.9602
8	1.2301	2.4591	1.9990
9	1.2687	2.5787	2.0325
10	1.3059	2.6931	2.0622
11	1.3415	2.8019	2.0887
12	1.3760	2.9063	2.1121
13	1.4092	3.0068	2.1337
14	1.4414	3.1037	2.1533
15	1.4726	3.1975	2.1713
16	1.5032	3.2885	2.1877
17	1.5327	3.3765	2.2030
18	1.5615	3.4617	2.2170
19	1.5896	3.5453	2.2304
20	1.6172	3.6266	2.2426

Table 8: The hypotenuse values and their ratios

c	AC	CE	R_4=CE/AC
1	0.6797	2.9591	4.3539
2	1.2132	3.3717	2.7792
3	1.6618	3.7215	2.2394
4	2.0552	4.0289	1.9603
5	2.4103	4.3060	1.7865
6	2.7357	4.5602	1.6669
7	3.0394	4.7970	1.5783
8	3.3243	5.0192	1.5099
9	3.5923	5.2288	1.4555
10	3.8485	5.4285	1.4105
11	4.0928	5.6185	1.3728
12	4.3262	5.8016	1.3410
13	4.5516	5.9767	1.3131
14	4.7691	6.1460	1.2887
15	4.9797	6.3094	1.2670
16	5.1832	6.4688	1.2480
17	5.3809	6.6223	1.2307
18	5.5715	6.7718	1.2154
19	5.7592	6.9174	1.2011
20	5.9409	7.0599	1.1884

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